ARITHMETIC MEAN

Dr. Bibha Verma
PG Department of Zoology
Maharaja College Ara

PG Sem II, CC-6, Unit -III

Definition of Arithmetic Mean

The arithmetic mean is often known simply as the mean. It is an average, a measure of the center of a set of data. The arithmetic mean is calculated by adding up all the values and dividing the sum by the total number of values.

For example, the mean of 77, 44, 55 and 88 is 7+4+5+84=67+4+5+84=6.
If the data values are x1x1, x2x2, …, xn, then we have x¯=1n∑ni=1xix¯=1n∑i=1nxi,
where x¯x¯ is a symbol representing the mean of the xi values.
This rearranges to give the useful result

nx¯=∑i=1nxi, nx¯=∑i=1nxi,

that is, the arithmetic mean is the number x¯x¯ for which having nn copies of this number gives the same sum as the original data. So the sum of a set of numbers in some sense "averages" them.
If the data are grouped, with fifi occurrences of the value xi for i=1i=1, 22, …, n, then their mean is given by

x¯=∑ni=1fixi∑ni=1fi,x¯=∑i=1nfixi∑i=1nfi,

where the numerator is the sum of all of the xi values and the denominator is the total number of values.
The arithmetic mean is sensitive to outlier values.

Properties of Arithmetic Mean

Some important properties of the arithmetic mean are as follows:

- The sum of deviations of the items from their arithmetic mean is always zero, i.e. $\sum(x - X) = 0$.
- The sum of the squared deviations of the items from Arithmetic Mean (A.M) is minimum, which is less than the sum of the squared deviations of the items from any other values.

- If each item in the arithmetic series is substituted by the mean, then the sum of these replacements will be equal to the sum of the specific items.

Merits of Arithmetic Mean

- The arithmetic mean is simple to understand and easy to calculate.
- It is influenced by the value of every item in the series.
- A.M is rigidly defined.
- It has the capability of further algebraic treatment.
- It is a measured value and not based on the position in the series.

Demerits of Arithmetic Mean

- It is changed by extreme items such as very small and very large items.
- It can rarely be identified by inspection.
- In some cases, A.M. does not represent the original item. For example, average patients admitted to a hospital are 10.7 per day.
- The arithmetic mean is not suitable in extremely asymmetrical distributions.

Arithmetic Mean Example

The Arithmetic means utilizes two basic mathematical operations, addition and division to find a central value for set of values. If you wanted to find the arithmetic means of the runs scored by Virat Kohli in the last few innings, all you would have to do is sum up his runs to obtain sum and then divide it by the number of innings. For example;

Innings	1	2	3	4	5	6	7	8	9	10
Runs	50	59	90	8	106	117	59	91	7	74

The arithmetic mean of Virat Kohli's batting scores also called his Batting Average is;

Sum of runs scored/Number of innings = 661/10

The arithmetic mean of his scores in the last 10 innings is 66.1. If we add another score to this sum, say his 11th innings, the arithmetic mean will proportionally change. If the runs scored in 11th innings are 70, the new average becomes;

$$\frac{661 + 70}{10 + 1} = \frac{731}{11} \quad 66.45$$

The average is a neat tool, but it comes with its set of problems. Sometimes it doesn't represent the situation accurately enough. I'll show you what I mean. Let's take the results of a class test for example. Say there are 10 students in the class and they recently gave a test out of 100 marks. There are two scenarios here.

First: 50, 53, 50, 51, 48, 93, 90, 92, 91, 90

Second: 71, 72, 70, 75, 73, 74, 75, 70, 74, 72

Practice Problems

1: Find the mean driving speed for 6 different cars on the same highway.

66 mph, 57 mph, 71 mph, 54 mph, 69 mph, 58 mph.

2: The Schauer family drove through 4 midwestern states on their summer vacation. Gasoline prices varied from state to state. What is the mean gasoline price?

$1.79, $1.61, $1.96, $2.08

3: Find the mean swimming time rounded to the nearest tenth:

2.6 min, 7.2 min, 3.5 min, 9.8 min, 2.5 min

4: A marathon race was completed by 5 participants in the times given below. What is the mean race time for this marathon?

2.7 hr, 8.3 hr, 3.5 hr, 5.1 hr, 4.9 hr

Solutions & Answer Key:

1: 66 + 57 + 71 + 54 + 69 + 58 = 375

$$\begin{array}{r} 62.5 \\ 6\overline{)375.0} \end{array}$$

Answer: The mean driving speed is 62.5 mph.

2: \$1.79 + \$1.61 + \$1.96 + \$2.08 = \$7.44

$$\begin{array}{r} \$1.86 \\ 4\overline{)\$7.44} \end{array}$$

Answer: The mean gasoline price is \$1.86.

3: 2.6 + 7.2 + 3.5 + 9.8 + 2.5 = 25.6

$$\begin{array}{r} 5.12 \\ 5\overline{)25.60} \end{array}$$

Answer: The mean swimming time to the nearest tenth is 5.1 min.

4: 2.7 + 8.3 + 3.5 + 5.1 + 4.9 = 24.5

$$\begin{array}{r} 4.9 \\ 5\overline{)24.5} \end{array}$$

Answer: The mean race time is 4.9 hr

References:

https://brilliant.org/wiki/arithmetic-mean/
https://undergroundmathematics.org/glossary/arithmetic-mean
https://www.toppr.com/guides/maths/data-handling/arithmetic-mean/
https://whatis.techtarget.com/definition/arithmetic-mean
https://www.superprof.co.uk/resources/academic/maths/probability/normal-distribution/arithmetic-mean-problems.html#chapter_exercise-1
https://www.biologydiscussion.com/biometry/top-3-kinds-of-mathematical-average-biometry/61075
https://en.wikipedia.org/wiki/Arithmetic_mean
http://www.biostathandbook.com/central.html
https://www.tutorialspoint.com/statistics/arithmetic_mean.htm